ENTRANCE EXAMINATION PROGRAM

FOR PHYSTECH SCHOOL OF AEROSPACE TECHNOLOGY MATHEMATICS AND MECHANICS COMPETITIVE GROUP

FOR APPLICANTS ENTERING PHD PROGRAMS

The entrance examination is conducted in the form of an interview.
The interview consists of two parts:

- interview on the content of the final qualifying work written by the applicant at the end of the specialist's or master's degree - in accordance with Part I of this Program;
- interview on general theoretical questions of the chosen specialty- in accordance with Part II of this Program.

PART I

Questions on the final qualifying work of the applicant

(Master or specialist's degree)

1. Main provisions.
2. Novelty.
3. Relevance.

Specialty 1.1.9.

Fluid, Gas and Plasma Mechanics

PART II

1. General concepts of continuum mechanics

1.1. The concept of a continuum. Microscopic, statistical and macroscopic phenomenological methods for describing the properties, interactions and movements of material media.

1.2. Euler-Lagrange variables. Inertial and non-inertial frames of reference in Newtonian mechanics.
1.3. Multicomponent mixtures. Diffusion flows. Continuity equations in the Euler form for multicomponent mixtures.
1.4. The rate of change in the characteristics of a liquid particle and liquid volume. Integral record of conservation laws. Mass conservation, continuity equation in Euler-Lagrange variables.
1.5. Law of conservation of momentum. Mass and surface forces. Properties of surface forces. Cauchy's formula. Equation of motion in stresses. Euler's equations. Barotropism.
1.6. Heat flow equation. Heat flux vector. Differential equations of energy and heat influx. Fourier's law of heat conduction. Various particular processes: adiabatic, isothermal, etc.
1.7. Reversible and irreversible processes. Perfect gas. Carnot cycle. The second law of thermodynamics. Entropy and absolute temperature. Uncompensated heat and entropy production. Dissipation inequality, Gibbs identity. Dissipative function. Basic macroscopic mechanisms of dissipation. The concept of the Onsager principle. Equations of state. Thermodynamic potentials of two-parameter media.
1.8. Law of conservation of angular momentum. Symmetry of the stress tensor.
1.9. Vortex theorems. Thomson's circulation theorem. Lagrange's theorem on the conservation of potentiality. Cauchy-Lagrange and Bernoulli integrals.
1.10. Potential movements. Potential properties. Statement of problems about the flow around bodies. Flow around a ball.
1.11. Application of methods of the theory of analytic functions of a complex variable for solving plane problems of hydrodynamics and aerodynamics: concepts of stream function, complex potential. Uniform flow, source, vortex point. Flow around bodies. Stationary fluid flow around a circular cylinder and airfoil.
1.12. Method of conformal mappings. Hydrodynamic reactions to moving bodies. Blasius-Chaplygin formulas, Joukowski's theorem. Flow around the plate. Joukowski and Chaplygin's rule for determining circulation around wings with a sharp trailing edge. Unsteady flow around airfoils.

2. Fluid mechanics

2.1. Application of integral relations to finite volumes of a medium in steady motion. Theory of jet force and the ideal propeller theory.
2.2. Waves on the water. Kinematic and dynamic conditions on the free surface of a fluid. Linear waves. Waves in a basin of finite depth. Long, short waves.
2.3. Phase and group velocity. Wave dispersion. Energy transfer by progressive waves. Shallow water theory. Boussinesq and Korteweg-De Vries equations. Nonlinear waves. Soliton.
2.4. Equations of motion in Lagrange form. Gerstner waves.
2.5. The problem of the motion of bodies in an ideal fluid. Attached momentum and angular momentum. Added masses tensor. The equation of motion of the ball.
2.6. Strain rate tensor. Relation between stress tensors and strain rates. Newtonian fluid. Navier-Stokes equations. Media with other rheological relations.
2.7. Exact solutions of the equations of a viscous fluid: diffusion of a vortex layer and a vortex filament, Poiseuille flow, flow between two rotating cylinders. Flow in an elliptical pipe. The flow of a viscous fluid in a diffuser. Vortex diffusion.
2.8. Navier-Stokes equations in dimensionless form. Strouhal, Euler, Froude, Reynolds numbers. Similarity of hydrodynamic phenomena.
2.9. Approximate solutions for small Re numbers. Approximation of Stokes and Oseen. The problem of the motion of a sphere in a viscous fluid in the Stokes' formula.

2.10. Approximate solutions for large Re numbers. The boundary layer theory. Friedrich's example.
2.11. Laminar boundary layer. Blasius problem. Integral relations and approximate methods based on their use in the theory of a laminar boundary layer. The phenomenon of separation of the boundary layer. Boundary layer stability. Heat exchange with flow based on the theory of the boundary layer.
2.12. Hydraulic jumps in the shallow water theory. Relationship between shallow water theory and gas dynamics.
2.13. Influence of compressibility on the shape of stream tubes in steady motion. Elementary theory of the Laval nozzle.
2.14. Orr-Sommerfeld equation for plane-parallel viscous flows.
2.15. Squire's theorem. Inviscid instability. Rayleigh's theorems.
2.16. Developed turbulence. Averaging. Reynolds experience. Reynolds equations. Closure problem. The simplest closure methods. Energy approach. Dissipation and energy exchange between mean and turbulent motions. Semiempirical theories of turbulence.

3.Gas dynamics
3.1. One-dimensional gas dynamics. Propagation of small perturbations in a compressible fluid. Wave equation. Sound speed.
3.2. Doppler effect. Mach cone. Method of characteristics. Riemann invariants. Problems of a piston and a strong explosion in a gas.
3.3. The occurrence of jumps. Solutions with a gap. Burger’s equation as a model equation. Application of integral conservation laws. Shock waves in gas dynamics. Wave breaking effect. Hugoniot adiabatic.
3.4. Plane stationary supersonic gas flows. Prandtl-Meyer expansion fan. Oblique shock wave. Supersonic gas flow past a wedge and a cone. The concept of gas flow around bodies with a detached shock wave. Linear theory of flow past thin airfoils and bodies of revolution. Flows with hypersonic speeds. Newton's law of resistance.

4. **Detonation and combustion**
4.1. Theory of detonation and combustion waves. Jouguet's rule and its justification. The problem of the strong discontinuity structure. Qualitative description of the solution to the problem of the decay of an arbitrary discontinuity.
4.2. Calculation of combustion temperature and composition of combustion products. Fundamentals of chemical kinetics. Classification of chemical reactions. The law of mass action. Reaction rate constants. Activation energy, reaction order.
4.3. The rate of a nonisothermal adiabatic reaction. Flame propagation in a homogeneous fuel-air mixture. The problem of the normal flame front. Concentration limits of flame propagation. Flame propagation in a turbulent flow.
4.4. Diffusion combustion of gases. Combustion flame structure, frontal combustion model. Detonation combustion, Chapman-Jouguet detonation.

5. **Plasma Physics**
5.1. Electromagnetic field. Maxwell's equations in emptiness. Interaction of an electromagnetic field with conductors. Lorentz force. The law of conservation of total charge. Ohm's law. Media with ideal conductivity. The Umov–Poynting vector and equation. Joule heating. Momentum and heat flux equations for a conducting medium.
5.2. Magnetohydrodynamics equations. Conditions for the freezing of a magnetic field into a medium. The concept of polarization and magnetization of liquids.

5.3. Features of measurements in aerodynamics. Instruments for measuring total and static pressure, nozzles for measuring the direction of speed. Toepler's method, a shadow method for visualizing flows. Holographic interferometry. Thermocouples, influence of radiation on measurement accuracy. Optical and spectral methods for measuring temperature. Temperature sensitive ink.

6. Devices and Methods of Measurement

6.1. Laser Doppler velocity meter. Phase Doppler method for measuring particle sizes and velocities. PIV method. CARS method, laser -induced fluorescence method. Analog and digital methods for random processes treatment.

6.2. Analog and digital instruments for spectral measurements, correlations and probability distributions. Error in measuring the mean value and variance of a random signal due to the finiteness of the averaging time. Sampling frequency and quantization step in analog-to-digital conversion. Thermal anemometer, turbulence measurement.

6.3. Basic and derived units of measurement. Systems of units in measurements. Classification of systems of units. The dimension of a physical quantity. Dependent and independent dimensions. The Buckingham π theorem.

6.4. Elements of underground hydrodynamics. The concept of porosity and permeability. Darcy's law. Phase relative permeabilities. The Buckley-Leverett problem.

6.5. System of differential equations of underground fluid dynamics. Unsteady gas filtration. Examples of exact self-similar solutions.

References

1. Валландер С.В. Лекции по гидроаэромеханике: Учеб. Пособие/ Под ред. Н.Н. Полякова.-2-е изд. — СПб..: Изд-во. С-Петерб. ун – та, 2005. — 304 с.
2. Седов Л.И. Механика сплошной среды. Т. I. 4-е изд. — М.: Лань, 2004, т.1 536с, и т2. —584 с.
3. Кочин Н.Е., Кибель И.А., Розе Н.В. Теоретическая гидромеханика. Ч. I, II. М.: Физматгиз, 1963.
4. Ландау Л.Д., Лифшиц Е.М. Гидродинамика. 3-е изд. М.: Наука, 1986.
5. Прандтль Л. Гидроаэромеханика. РХД, 2000.
6. Уизем Дж. Линейные и нелинейные волны. М.: Мир, 1977.

Specialty 1.1.8.

Mechanics of Deformable Rigid Body

PART II

1. Deformed state theory. Displacement vector. Tensor of small deformations and small rotations. Principal invariants of the small strain tensor. Saint-Venant's compatibility conditions. Strain-rate tensor.
2. Stress state theory. Mass and surface forces. Cauchy formula. Stress tensor. Symmetry of the stress tensor. Main invariants of the stress tensor. Mohr pie chart.
3. Continuity equation. Equations of equilibrium and motion. The law of conservation of angular momentum and the symmetry of the stress tensor.

4. Thermodynamics of a deformable body. First law of thermodynamics (energy balance axiom). Clausius-Duhem inequality. Dissipative processes.
5. Elastic body. Expression for the stress tensor in terms of the thermodynamic potential derivative (internal energy, Helmholtz free energy) with respect to the strain tensor. Generalized Hooke's law for a linear thermoelastic material. Reducing the number of elastic constants. Connection between elastic constants of an isotropic elastic medium. Isothermal and adiabatic moduli of elasticity. General theorems of elasticity theory (uniqueness, reciprocity, minimum strain energy). Castigliano's theorem.
6. Statement of mathematical problems of the linear theory of elasticity in displacements and stresses. Semi-reverse method à la Saint-Venant. Variational statements of problems in the theory of elasticity and fundamentals of direct methods (Ritz, Bubnov-Galerkin).
7. The simplest problems of the theory of elasticity: deformation of a thick-walled pipe under the influence of internal and external pressures, torsion of circular cross section bar, stretching of a bar under its own weight, equilibrium of a plate element, thermal stresses in a hollow sphere.
8. Plane problems of the theory of elasticity. Plane stressed and plane deformed states. Stress function. Method of the theory of functions of a complex variable in a plane problem. Concentrated force acting on the boundary of an elastic half-plane.
9. The concept of the Green's function. Elementary solution of the first and second order. Statement of Hertz's contact problem.
10. Bending of beams. Internal shear force and internal bending moment. Plotting transverse force and bending moment diagrams. Plane section hypothesis. Geometric characteristics of sections in bending. Normal and shear stresses in bending. Zhuravsky's formula. Differential equation of the bending axis of the beam.
11. Rod systems. Statistically determined and statically indeterminate systems. Methods for calculating trusses and frames.
12. Torsion of rods with an elliptical cross section. Deplanation.
13. Stability of rods. The concept of sustainability. Energy method in stability analysis. Euler's formula and its application.
14. Theoretical strength. Basic theories of strength. Brittle and quasi-brittle fracture. Griffith's energy criterion and Irwin's force criterion for crack propagation in an elastic body. Cherepanov-Rice invariant integral.
15. Ideal plasticity. Isotropic and kinematic hardening. Plastic strain tensor. Deformation theory of plasticity. The concept of simple and complex loading (deformation). Statement of problems in the framework of small elastic-plastic deformations. Simple loading theorem. Uniqueness theorem. Method of elastic solutions. Method of variable elasticity parameters. Unloading theorem. Hollow sphere made of ideally plastic material under internal pressure.
16. Theory of plastic flow. Associated plastic flow rule. Tresca-Saint-Venant, von Mises, Coulomb-Mohr, Drucker-Prager criteria of fluidity.
17. Plane problem of flow theory. The concept of slip lines and their properties. Henky integrals. Strip compression between rough slabs.
18. Linear theory of viscoelasticity. Boltzmann's superposition principle. Maxwell and Voigt model. Relaxation and creep kernels, relaxation and creep functions. Temperature-time analogy. Methods for solving problems of the linear theory of thermoviscoelasticity: numerical methods, methods using Laplace-Carson transforms; methods based on the consequences of simple loading and simple deformation theorems.
19. Two types of waves in an elastic medium. Core speed of sound. Kinematic and dynamic conditions on the discontinuity surface.

References

1. Феодосьев В.И. Сопротивление материалов: Учеб. для вузов. 10-е изд., перераб. и

доп.
- М.: Изд-во МГТУ им. Баумана, 1999. - 592 с.
2. Лейбензон Л.С. Курс теории упругости. Москва Ленинград : ОГИЗ Гос. изд-вотехн.-теорет. лит., 1947.
3. Ильюшин А.А. Пластичность, ГТТИ, 1948 .
2. Седов Л.И. Механика сплошной среды. Т. I. 4-е изд. - М.: Лань, 2004, т.1 536с, и т2. - 584 с.
1. Ильюшин А.А., Победря Б.Е. Основы математической теории термовязкоупругости.М.: «Наука», 1970.
2. Работнов Ю.Н. Механика деформируемого твердого тела. - М.: Наука, 1988. - 712 с.
3. Тимошенко С. П., Гудьер Дж. Теория упругости: Пер. с англ./Под ред. Г. С. Шапиро. - 2- изд. - М.: Наука. Главная редакция физико-математической литературы, 1979, 560 с.
4. Кондауров В.И., Фортов В.Е. Основы термомеханики конденсированной среды. М.:Изд-во МФТИ. 2002.
5. Москвитин В.В. Сопротивление вязкоупругих материалов (применительно к зарядам на твердом топливе). - М.: изд-во «Наука», Главная редакция физико-математическойлитературы, 1972 г.
6. Малинин Н.Н. Прикладная теория пластичности и ползучести. Учебник для студентоввузов. Изд. 2-е, перераб. и доп. - М.: Машиностроение, 1975

Specialty 1.1.7.
Theoretical Mechanics, Machine Dynamics

PART II

1. Axiomatics of classical mechanics. Point kinematics

1.1. Postulates of classical mechanics. Newton's laws. Inertial reference systems.

1.2. Galilean transformations. The concept of invariance and covariance of the equations of mechanics.

1.3. Trajectory, speed, acceleration. Natural (accompanying) trihedron. Decomposition of velocity and acceleration in the axes of a trihedron. Curvilinear coordinates of a point. Decomposition of the velocity and acceleration of a point in the local basis of curvilinear coordinates. Lamé coefficients.

2. Kinematics of a rigid body (kinematics of frames of reference)

2.1. Rigid body. Arrangement of body movement into translational movement and rotation (movement with a fixed point).

2.2. Methods for specifying the orientation of a rigid body: Euler angles, direction cosine matrices. Quaternion algebra. Quaternion way of specifying the orientation of a rigid body (attached display). Rodrigues-Hamilton parameters. Quaternion formulas for adding rotations.

2.3. Kinematic equations of rotational motion of a rigid body in quaternions (Poisson's equations). Euler's theorem on finite rotation of a rigid body with a fixed point. Angular velocity and angular acceleration of a rigid body. Distribution of velocities and accelerations in a rigid body (Euler and Rivals theorems). Motion of a free rigid body. Kinematic screw.

3. Kinematics of complex motion

3.1. Addition of velocities and accelerations of points in a complex motion. Calculation of the angular velocity and angular acceleration of a body in a complex motion.

4. General theorems of dynamics

4.1. Definitions: external and internal forces, momentum (quantity of motion), angular momentum (kinetic moment, quantity of motion moment), kinetic energy, center of mass, moment of force, elementary work and power of force.

4.2. König's theorems for kinetic energy and angular momentum. Theorems on the change of momentum, angular momentum and kinetic energy in inertial frames of reference. Conservative systems, law of conservation of energy.

4.3. Non-inertial reference frames, inertial forces. Basic theorems of dynamics in non-inertial reference frames.

5. Movement of a material point in the central field

5.1. Conservation laws (first integrals of motion). Binet equation. The field of universal gravitation. Conic sections equations. Two-body problem. Kepler's laws.

6. Dynamics of systems of variable composition

6.1. The concept of a system of variable composition and its mathematical model.

6.2. Theorems on the change in momentum and angular momentum for systems of variable composition. Meshchersky equation. Jet propulsion. Tsiolkovsky equation.

7. Rigid body dynamics

7.1. Mass point geometry. Tensor of inertia and ellipsoid of inertia rigid body. Principal axes of inertia. Transformation of the inertia tensor during rotation and parallel translation of the axes. The Huygens–Steiner theorem for the inertia tensor.

7.2. Kinetic moment and kinetic energy of a rigid body. Euler’s dynamic equations. Euler’s case; first integrals of motion; geometric interpretations of Poinsot and McCullagh.

7.3. Motion of a dynamically symmetric body in the case of Euler; parameters of free regular precession. The Lagrange case; first integrals of motion.

7.4. Formula for the moment supporting the forced regular precession of a dynamically symmetric rigid body.

8. Lagrangian mechanics

8.1. The concept of mechanical connection. Classification of connections. virtual movements. General dynamics equation for a system of material points with ideal connections.

8.2. Configuration manifold of a holonomic system with a finite number of degrees of freedom. Generalized coordinates. Lagrange's equations. Generalized forces. Potential, gyroscopic, dissipative forces. The criterion of the potential of forces.

8.3. Lagrange equations in case of potential forces; Lagrange function (Lagrangian system).

8.4. Lagrange equations in non-inertial reference frames. Properties of the Lagrange equations: covariance, non-degenerate distribution (reduction to normal Cauchy form).

8.5. Kinetic energy structure. First integrals of Lagrangian systems: cyclic integrals, generalized energy integral (Painlevé-Jacobi integral).

9. Equilibrium, stability, movement near a stable equilibrium position

9.1. Determining equilibrium position. Equilibrium conditions for a system with ideal connections (principal of virtual work). Equilibrium conditions for holonomic systems (in terms of generalized forces).

9.2. Definition of stability, asymptotic stability and instability of the equilibrium position. Theorems of Lyapunov's direct method for autonomous systems: Lyapunov's theorems on stability and asymptotic stability, Chetaev instability theorem, Barbashin-Krasovskii's theorem on conditions of asymptotic stability and instability.

9.3. The Lagrange–Dirichlet theorem on the stability of the equilibrium of conservative mechanical systems. Conditions for the instability of conservative systems with respect to the quadratic part of the potential energy. The concept of bifurcation. Cases of loss of stability for systems with a potential depending on a parameter. Influence of gyroscopic and dissipative forces on the stability of equilibrium. A theorem on the asymptotic stability of strictly dissipative systems.

9.4. Lyapunov's first method for studying stability. Lyapunov's theorem on stability with respect to linear approximation. Routh-Hurwitz criterion (without proof). Two buckling scenarios: divergence and flutter.

9.5. Small fluctuations of conservative systems near a stable equilibrium position. Frequency equation. Principal (normal) coordinates. Common decision. Multiple roots case.

9.6. Forced vibrations of a linear stationary system under the action of harmonic forces. Frequency characteristics. Resonance phenomenon. Response of a linear stationary system to non-harmonic action.

10. Hamilton's equations, variational principles, integral invariants

10.1. Hamilton variables. Hamilton function. Canonical equations of Hamilton. Legendre transformation of Lagrange's equations into Hamilton's equations. Hamilton's function for a conservative system.

10.2. First integrals of Hamiltonian systems. Poisson brackets. Jacobi–Poisson theorem. Lowering the order of Hamilton's equations in the case of cyclic coordinates and for generalized conservative systems. Whittaker equations.

10.3. Hamilton's action principle. A variation of the action according to Hamilton. Hamilton's variational principle.

10.4. Lagrangian transformation under change of coordinates and time. Emmy Noether's theorem.

10.5. Poincaré–Cartan and Poincaré integral invariants. Inverse theorems of the theory of integral invariants. Liouville's theorem on the invariance of the phase volume of a Hamiltonian system. Lee Hwa-Chung theorem on integral invariants of the first order of Hamiltonian systems.

11. Canonical transformations. Hamilton–Jacobi equation

11.1. Canonical transformations. Local criterion of canonicity. Canonicity criterion in terms of generating functions.

11.2. Transformations that allow $(q,\tilde{q})$ – description (free transformations). Rules for the transformation of Hamiltonians under canonical transformations. Phase flow of Hamiltonian systems as a one-parameter family of canonical transformations.

11.3. Hamilton–Jacobi equation. The complete integral of the Hamilton–Jacobi equation and its use in the problem of integrating the motion equations of a Hamiltonian system. Cases of separation of variables.

References

1. Айзерман М.А. Классическая механика: учебное пособие. – 3-е изд. – М.: Физматлит,2005. – 380 с.
2. Гантмахер Ф.Р. Лекции по аналитической механике. – 3-е изд. – М.: Физматлит , 2001.
3. Журавлёв В.Ф. Основы теоретической механики. – 2-е изд. – М.: Физматлит, 2001; 3-еизд. – М.: Физматлит, 2008.
4. Маркеев А.П. Теоретическая механика. Учебник для университетов. Изд. 3-е, исправленное – М.: Издательство «Регулярная и хаотическая динамика». 2001. – 592 с.
5. Яковенко Г.Н. Краткий курс теоретической механики. – М.: БИНОМ. Лаборатория знаний, 2006.
6. Яковенко Г.Н. Краткий курс аналитической динамики. – М.: БИНОМ, 2004. – 238 с.